

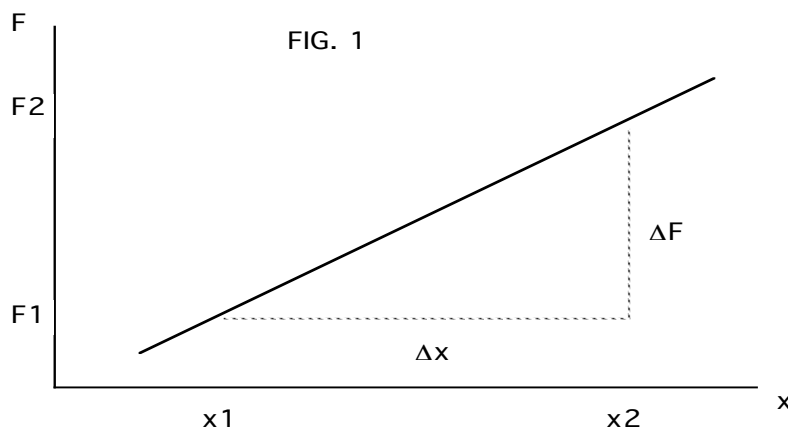
Hooke's Law: Simple Harmonic Motion

I. Introduction

The force exerted by a stretched spring, when its elastic limit has not been exceeded, was found by Robert Hooke, in 1676, to be proportional to its elongation. The force and the stretching of the spring are linearly related; equal increases in the length of the spring are accompanied by equal increases in the force exerted by the spring. The relationship between the magnitude of the force, F , exerted by the spring, and the elongation, x , of the spring beyond its unstretched length is given by

$$F \sim x \quad \text{or} \quad F = kx$$

the graph for which is shown in Fig1 below. The physical slope of the graph, $\frac{\Delta F}{\Delta x}$, is equal to the "force constant", k , of the spring.



This knowledge of the behavior of a stretched spring, known as Hooke's law, makes possible the analysis of the motion which results when a mass, fixed at the lower end of a vertically hanging spring, vibrates up and down in the earth's gravitational field. In Fig 2, the mass M is shown hanging in the equilibrium position. When displaced from the equilibrium position at $x=0$ and released, it oscillates up and down in simple harmonic motion. The displacement is periodic and given by

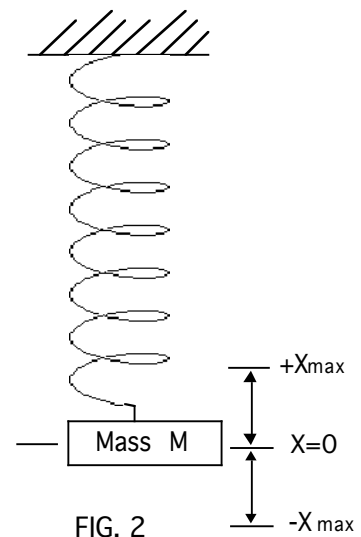
$$x(t) = A \sin \left(2\pi \frac{t}{T} + \theta \right)$$

where A is the "amplitude", or maximum x displacement, T is the "period", or time for a single cycle, and θ is the "initial phase".

For simple harmonic motion, the period T is independent of the amplitude but does depend on the stiffness of the spring (force constant k) and the inertial mass M where

$$T = 2\pi \sqrt{\frac{M}{k}}$$

In this experiment, the objective is to determine the force constant, k , two ways - by investigating Hooke's law and by investigating the period of oscillation.



II. Data Collection

a) Method 1

For the Hooke's law, or static method, hang a series of weights on the spring and measure the resulting vertical positions. Please gather about 8 to 10 data pairs of mass and resulting position. Choose appropriate masses to stretch the spring so that you do not exceed the elastic limit and permanently deform the spring. For the typical spring supplied, limit your weights to less than about 1000 grams. Either a vertical scale or the ultrasonic position detector may be used to measure the vertical position.

The final product should be a graph like Fig.1. A straight line confirms Hooke's law and the slope of the line is the spring constant k . The true origin shown in Fig.1 can not be found and is not needed for the experiment. Some springs have a "built in tension" where a significant force is required before the windings on the spring even begin to separate, or they come with some weight already attached. So, do not try to display the true origin, but only plot the force (weight added) versus the vertical position.

When plotting the graph, please convert the mass to force units in Newtons and the position units to meters.

b) Method 2

For the period method, or dynamic method, apply various masses to the spring and measure the resulting periods. The inertial mass M consists of the mass you add, m , the mass of the attached hanger, m_H and (according to theory) one third the mass of the spring, m_S . That is,

$$T = 2\pi \sqrt{\frac{m + m_H + \frac{1}{3} m_S}{k}} \quad \text{or} \quad T^2 = \frac{4\pi^2}{k} m + \frac{4\pi^2}{k} (m_H + \frac{1}{3} m_S)$$

Therefore, a straight line graph of the square of the period (T^2) plotted as a function of the mass (m) will have

$$\text{slope} = \frac{4\pi^2}{k} \quad \text{and} \quad \text{intercept} = \frac{4\pi^2}{k} (m_H + \frac{1}{3} m_S)$$

Please gather 8 to 10 data pairs of period T and added mass m . To find a more accurate value of the period, time over 20 oscillations with a stop watch or use the ultrasonic position detector with DataLoggerPro to record the oscillatory motion and read the period from the computer screen.

III. Results and Conclusions

a) Method 1

Using Graphical Analysis software, plot a graph of F vs x . Determine a static value of the spring constant and its uncertainty from the slope of the straight line using the linear fit.

Report the result and its uncertainty with appropriate significant digits.

b) Method 2

Plot a graph of T^2 vs. m . Using the value of the slope from the linear fit, solve for the spring constant k . (note: The slope is not equal to k .) Report the result and its uncertainty with appropriate significant digits.

Questions: Do the two values of the spring constant k agree when you include the uncertainties?
What is the percent difference between the two values of k ? (B.L.Hurst 2006)